The flashing ratchet and transport of matter

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Overview

We study the flashing ratchet model of a Brownian motor, which consists in cyclical switching between the Fokker-Planck equation with an asymmetric ratchet-like potential and the pure diffusion equation. We show that the motor really performs unidirectional transport of mass, for proper parameters of the model, by analyzing the attractor of the problem and the stationary vector of a related Markov chain.

Brownian motors

Brownian motors are nano-scale or molecular devices by which thermally activated processes (chemical reactions) are controlled and used to generate directed motion in space and to do mechanical or electrical work. These tiny engines operate in an environment where viscosity dominates inertia, and where thermal noise makes moving in a specific direction as difficult as walking in a hurricane: the forces impelling these motors in the desired direction are minuscule in comparison with the random forces exerted by the environment. Because this type of motor is so strongly dependent on random thermal noise, Brownian motors are feasible only at the nanometer scale.

In biology, many protein-based molecular motors in the cell may in fact be Brownian motors. These molecular motors convert the chemical energy present in ATP into mechanical energy. One example of a Brownian motor would be an ATPase motor that hydrolyzes ATP to generate fluctuating anisotropic energetic potentials. The anisotropic potentials along the path would bias the motion of a particle (like an ion or polypeptide); the result would essentially be diffusion of a particle whose net motion is strongly biased in one direction. The translocation of the particle would only be loosely coupled to hydrolysis of ATP.

The dynamics and activity of Brownian motors are current topics of study in theoretical and experimental biophysics. Brownian motors are sometimes modeled using the Fokker-Planck equation or with Monte Carlo methods. Many researchers are presently engaged in understanding how molecular-scale motors operate in environments with non-negligible thermal noise. The thermodynamics of such motors is constrained by the ramifications of the Fluctuation Theorems, Pumping Quantization Theorems, and Pumping-Restriction Theorems. (Wikipedia)

Parrondo's paradox

Given two games, each with a higher probability of losing than winning, it is possible to construct a winning strategy by playing the games alternately.

Example of Parrondo's paradox

Game A consists of tossing a biased coin (coin 1) that has a probability (p1) of winning of less than half, so it is a losing game. Let $p = 1/2 - e$, where e, the bias, can be any small number, say 0.005. Game B consists of playing with two biased coins. The rule is that we play coin 2 if our capital is a multiple of 3 and play coin 3 if it is not. This means that, on average, coin 3 would be played a little more often than coin 2. If we assign a poor probability of winning to coin 2, such as $p = 1/10 - e$, then this would outweigh the better coin 3 with $p = 3/4 - e$, making game B a losing game overall. Thus both A and B are losing games. If we play two games of A followed by two of B and so on, we get a rapid increase in capital — this is Parrondo's paradox. What is even more remarkable is that when games A and B are played randomly, with no order in the sequence, this still produces a winning expectation.

Flashing ratchet

- *Pure Fokker-Planck — no transport*
- *Pure diffusion — no transport*
- *Periodic switching between Fokker-Planck and diffusion — unidirectional transport*

However, the mechanism seems to be quite different from the Parrondo's one.

Questions?

Thank you!